Electromagnetic Coupling Circuit Model of a Magnetic Near-Field Probe to a Microstrip Line

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Abstract—Electromagnetic (EM) injection experiments require an accurate and quantitative knowledge of the voltage effectively coupled to a target line or circuit in order to predict disruptive behavior or sensitivity of digital IC circuits to EM threats. To answer this question we derive here a complete quantitative model of the coupling of our magnetic probe to a microstrip line. The novelty of this model is to consider the coupling by analogy with a transformer and then to deduce the corresponding mutual inductance as a function of probe to target relative positions. Its inputs are S-parameter measurements of the actual probe coupled to a 50 Ω microstrip line and its output is an electric equivalent circuit that can be implemented in any circuit simulator. Validity of the model extends up to GHz frequencies.

Index Terms—Electromagnetic injection, Electromagnetic coupling model, Electrical circuit model, Magnetic near-field probe.

I. INTRODUCTION

Electromagnetic injection (EMI) is a technique that has proven its effectiveness in studying the susceptibility of integrated circuits (ICs) [1], [2] and has found recently applications in fault-injection combined with side-channel attack as a new physical technique to break a cryptosystem [3]. Such new usages of EMI drive optimizations of the attacks that in practice rely on the optimization of the probe and on an enhanced understanding of its coupling with the possible on-chip receivers that are known to be mostly the long supply voltage lines and eventually the long signal lines [4].

The EMI optimization is two-fold. The probes must be optimized to deliver an intense and localized magnetic field that is most efficiently coupled to on-chip long lines. In a previous study an injection near-field probe has been improved considering only its radiative properties [5]. We then intend in this work to understand and quantify as accurately as possible the coupling between the probe and the IC. In fact this is now theoretically possible using 3D electromagnetic softwares, but such a method is quite long and tedious. Another possibility is to calculate analytically the coupling using formalisms like those of Ref. [6], but the subtle details of the probe geometry seems impossible to account for. Here we chose to derive an equivalent circuit model that will be probe dependent but very accurate and include by essence the probe as a whole. It fits our final goal that is to build an electrical circuit model suitable for implementation in classical circuit simulators. By the way EMI attacks could be taken into account at the early stage of the IC layout design and eventually one can then prevent against such EMI threats. Notice that the method employed can be easily transposed to another magnetic probe design.

The §II of the paper recalls the magnetic ferrite probe design and optimization and propose an equivalent electrical circuit model validated against S-parameters measurements. The §III details the near-field probe to line coupling and supports the coupling model via mutual inductance, like in a ideal transformer. Optimization guidelines relative to probe-line relative position as well as number of loops wound around the ferrite core are given. The §IV deals with an application of the deduced electrical model to a realistic coupling situation with a pulse input.

II. THE PROBE

A. Geometrical Description

Let us first recall the geometrical parameters of the magnetic ferrite probe already designed and realized [5]. In that latter study we focused on the optimization of the field concentration at probe tip end and this was obtained with a ferrite core conically machined so as to exhibit a tip of a few tenths of µm radius (150 µm typical for practical realizations and 20 µm for simulation). The magnetic field is produced by winding N loops of 50 µm diameter Cu wire around a ferrite rod of 2 mm diameter made of MnZn alloy (see Fig. 1).

Owing to the magnetic guiding properties of the ferrite core the field generation and its guiding and concentration up to the interaction with the target are fully decoupled. By using CST Microwave Studio [7], we have optimized this probe following the two criteria of spatial resolution and injection efficiency. According to electromagnetic simulations, this geometrical configuration should provide a spatial resolution of ≲ 200 µm for a tip-sample distance of 20 µm together with a maximum field intensity that is proportional to N, the number

Fig. 1. Design view of the modeled ferrite with a conical cut.
of loops. This characteristic holds if the following conditions are satisfied: the field remains below the saturation field of the ferrite core, the frequency does not exceed $\approx 1 \text{GHz}$ and the loop count is not too high so as the parasitic capacitance between loops remains sufficiently low.

B. Electrical Model of the Probe

A better understanding of the probe-circuit interaction requires accurate characterization. This was done owing to $S$-parameters with the same methodology as in Ref. [1]. Focusing on $S_{11}$ in the frequency range of 1 MHz to 1 GHz allows to derive an electric model with lumped elements by means of the known transformation of $S$-parameters to $Z$-parameters. The proposed electrical SPICE model of the probe is given in Fig. 2.

![Fig. 2. Equivalent electrical circuit model of the magnetic probe and of the coupling to the line. The probe lumped equivalent circuit is given in the upper part with $R_w$, $L_w$, $R_f$, $L_f$ and $C_p$, the generator and 50 Ω load figuring the input signal fed into the probe. The coupling model is described by $M$ and $C_s$. The line is modeled by distributed $LC$ cells ended by input and output devices (50 Ω load, transistor, ...).](image)

This model is intended to replicate the electrical behavior of the sensor while being close to the physics and geometry of the probe. The wire connecting the $N$ loops wound around the ferrite core to the SMA connector is modeled by a resistance in series with an inductance whose values are set proportionally to their true length and known resistivity. The ferrite with its loops is accounted for by the $RLC$ circuit, $C_p$ corresponding to the inter-loop winding capacitance. The resistance $R_f$ that accounts for magnetic losses and the inductance $L_f$ are frequency dependent and they both come from the physical model of the real and imaginary part of the ferrite rod [8]. Figure 3 depicts the known evolution of $\mu_{\text{rod}} = \mu_{\text{rod}}' - j\mu_{\text{rod}}''$ of our MnZn ferrite rod in the entire frequency band of interest.

The real part of the relative permittivity of the ferrite rod, $\mu_{\text{rod}}'$, is the multiplicative factor entering the calculation of the coil inductance, $L_f(f)$. It is almost constant and maximum at low frequency and decreases rapidly beyond 200 MHz, thereby reducing the efficiency of the probe. In the same time the imaginary part, $\mu_{\text{rod}}''$, increases regularly inducing corresponding increasing losses, $R_f(f)$, as the frequency rises. The optimum frequency band for a magnetic sensor using this ferrite rod material is thus from 1 MHz to 200 MHz.

With this model in hands, an adjustment of the circuit element values to measurement data has been done. Values are given in Tab. I for three probes with different $N$. Figure 4 then gives the comparison, in magnitude and phase of $Z_{11}$, between the calculated probe model impedance and the corresponding measurement. The increase of $|Z_{11}|$ proportionally to the frequency arising together with a constant $90^\circ$ phase corresponds to an inductive behavior up to 200 MHz. The resonance occurring at 400 MHz indicates a change in the electrical behavior of the probe that can not anymore be assimilated to a perfect inductance. The excellent agreement between model and measurements, over three decades in frequency extending up to 1 GHz, validates the high capability of our model to model electrically this probe.

![Fig. 3. Real and imaginary part of the MnZn ferrite rod permeability.](image)

| TABLE I |
| PARAMETERS OF THE PROBE MODEL. |

<table>
<thead>
<tr>
<th>$N = 3$</th>
<th>$N = 6$</th>
<th>$N = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_w$ (Ω)</td>
<td>0.3</td>
<td>0.25</td>
</tr>
<tr>
<td>$L_w$ (nH)</td>
<td>42</td>
<td>38</td>
</tr>
<tr>
<td>$C_p$ (pF)</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$L_f$ @10 MHz (nH)</td>
<td>137</td>
<td>910</td>
</tr>
<tr>
<td>$R_f$ @10 MHz (Ω)</td>
<td>0.3</td>
<td>2</td>
</tr>
</tbody>
</table>

III. Coupling Model of a Near-Field Probe to a Line

Let us now consider the coupling between the probe and the circuit under test. Since long lines were identified as the most interesting targets of our injection application, we will consider here a known 50 Ω microstrip line on PCB. Note that in practice there is no loss of generality to consider only a 50 Ω microstrip line because the voltage coupled to a line is given by the Maxwell-Faraday law and the knowledge of the magnetic field produced by the probe [6]. As long as the emitted field does not depend of the target, a feature verified here because it is valid except in the extreme near-field region (Rayleigh or reactive near-field region [9]), the coupling will
be modified only according to the line geometry in terms of the area under the microstrip. It thus follows that a simple proportionality rule holds when changing the line geometry.

In the following we will study both the influence of \( N \) and the relative location of the probe with respect to the target line. An optimization of the probe is looked for in terms of spatial resolution and injection efficiency. Characterizations have been performed on a 50 \( \Omega \) characteristic impedance, 5 cm long and 500 \( \mu \)m wide microstrip PCB line. Magnetic probes with three different loop counts of \( N = 3 \), \( 6 \) and \( 12 \) have been fabricated and considered.

A. The Ideal Transformer Picture

Our probe is designed for magnetic coupling. The natural picture for the coupling of this probe to any target is thus the ideal transformer. The input of the transformer is the probe and its output is the target, here the line. Both are linked through a mutual inductance to be determined. Assuming that the probe is at port 1 and the line at port 2, one can characterize experimentally the system by measuring its \( S \)-parameters then subsequently converted to \( Z \)-matrix. As a consequence the mutual inductance \( M \) is directly obtained from the \( Z_{21} \) parameter by

\[
Z_{21} = jM\omega \quad \rightarrow \quad M = \frac{Z_{21}}{j\omega} \quad (1)
\]

The novelty and key parameter of our model lies in this \( M \) mutual inductance whose extraction is performed by measuring \( Z_{21} \) in the 1 MHz–1 GHz frequency band. This is done by connecting a network analyzer to our near-field injection bench (see Fig. 5). The probe position is adjusted in the three \( XYZ \) directions using motorized stages controlled by computer. Relative motions, including height and lateral displacement of the line with respect to the probe, are then considered to seek for optimal coupling conditions by maximizing \( M \) from \( Z_{21} \) measurements. Similarly one can compare probes with various characteristics at a given position with respect to the line.

\[\text{Fig. 4. Modulus and phase of the measured and modeled probe input impedance. The probe has } N = 3 \text{ loops.}\]

\[\text{Fig. 5. Experimental setup for near-field EMI. Probe motion across the line is shown.}\]

B. Extraction of \( M \)

In a first attempt we chose a probe positioning at height of 100 \( \mu \)m over the circuit with the probe tip slightly away from the line, just like it was drawn in Fig. 5. \( M \)-values were extracted over the whole 1 MHz–1 GHz frequency range for two probes differing by the number of loops, namely \( N = 3 \) and \( N = 12 \). A tremendous result of Fig. 6 is the constant \( M \)-values obtained at low frequencies for both probes. This emphasizes our model picture of an ideal transformer. The \( N = 3 \) probe exhibits a constant \( M = 0.9 \) nH up to 300 MHz while the \( N = 12 \) probe has a \( M = 2 \) nH constant value up to \( \approx 30 \) MHz. Beyond these limits a cutoff behavior is observed together with an overshoot that is more or less pronounced depending on \( N \). For sure at such high frequencies the transformer is no more ideal for two reasons. First the receptive line is no more a pure inductance, and second the capacitive coupling between the probe and the line increases with the frequency [10]. A simple model improvement was to include these two factors of non-ideality: the microstrip line is practically modeled using distributed \( LC \) elements [11], and the capacitive coupling is taken into account in the model by adding the fitted stray capacitance \( C_s \) between the probe and the line. The now complete electrical scheme of probe to line coupling is given in Fig. 2 and is used in the following for optimization as a function of loop count and position.

C. Coupling vs Loop Count

The mutual inductance being the key value of the coupling, we decide first to compare various probes on this sole criteria as a function of the lateral position with respect to the target line, the height being prescribed at \( h = 100 \) \( \mu \)m. For each probe at each position the \( M \)-value is obtained as an average over the lower frequencies like it was shown in Fig. 6. Results are given in Fig. 7 for three probes having \( N = 3, 6 \) and 12 loops around a ferrite core of 2 mm diameter. Important features appear on these plots:
First the coupling is found important even far from the target, but surprisingly it is minimum just over the line and maximum 2 mm away on both sides! This comes from the field emitted by the probe which is mainly vertical in our scheme and that must encompass the line to create an induced voltage by mean of Maxwell-Faraday law. As a result, our \( M \) determination, that is the convolution of the intrinsic resolution of the probe with the intrinsic resolution of the detecting line, is poorly spatially localized. Although the spatial resolution of the probe alone has been optimized (see §II), the whole system becomes less accurate because of a bad matching between emitter and receiver.

Second the coupling is found highly dependent of \( N \). Quantitatively it is almost proportional to \( N \) since the maximum values measured are \( M \approx 1.05, 1.8, \) and 3.1 nH for \( N = 3, 6 \) and 12 loops. However, this must be tempered with the strong decrease of the maximum usable frequency observed as \( N \) increases (see Fig. 6). Thus the choice of \( N \) is highly application dependent and must be put in accordance with the speed of the EMI excitation signal.

**D. Coupling vs Height**

Similarly the \( M \)-value is relevant to compare the coupling of one specific set of probe–line as a function of probe height. This has been done for several probes but only the results obtained with the \( N = 6 \) loops probe crossing the line at the three constant heights of \( h = 50, 100 \) and 200 \( \mu \)m are given in Fig. 8.

The height has a minor role on the injection efficiency. Indeed a very small decrease (less than 10\%) is observed when \( h \) is multipled by 4. One can imagine that the change in \( h \) used here is quite negligible as compared to the mm-scale resolution of the coupled probe–line system with the simultaneous consequence of a very weak influence on both the coupling resolution and efficiency.

**IV. Coupling Model Application**

As an illustration of our model usage, we calculate the EMI coupling signal on the supply voltage of a bipolar transistor in 0.25 \( \mu \)m BiCMOS technology. Namely the electrical scheme of Fig. 2 is used with a pulse signal at probe input. Simulation was performed using ADS with the excitation depicted in Fig. 9 and has a voltage excursion of \( \pm 5 \) V, a repetition frequency of 10 MHz, risetime and falltime of \( \tau_r = \tau_f = 3 \) ns. Also given in Fig. 9 is the voltage across the ferrite coil which is almost the derivative of the current in the probe, and therefore a quantity proportional to the expected voltage coupled to the target owing to the Maxwell-Faraday law.

The simulation is intended to illustrate a realistic supply voltage line in 0.25 \( \mu \)m BiCMOS technology. Both ends of the target line are therefore loaded by a voltage source with its capacitance at In port and by a bipolar connected as
an inverter at Out port. The Fig. 10 gives the calculated voltage at that port. One can observe that more than 10 mV peak-to-peak voltage excursion are induced at that point and superimposed to intrinsic oscillations brought by the internal bipolar dynamics. These values are not sufficient to perturb the internal logic of an IC but the excitation considered here was at least ten times lower than what is usually considered for that purpose [3]. This application example of our coupling model also shows that induced perturbations are not easily predictable: for instance, if the transistor and voltage source are replaced by 50 Ω loads the coupled signal is two times less in amplitude and does not exhibit rapid oscillations. In the future, it is thus expected that our model can help in identifying the inner mechanisms of EMI fault injection by accounting of the exact layout of logic ICs.

V. CONCLUSION

The coupling between a magnetic near-field probe and a line has been experimentally studied and modeled. The probe is built from a ferrite core machined in conical form with few wire loops wound to produce the magnetic field that is concentrated at ferrite tip end. This design was intended to optimize the injection efficiency and spatial resolution. The interaction between the probe and the line was experimentally characterized up to 1 GHz. Experiments were used to derive an original lumped-elements equivalent coupling-model suitable for an implementation in electrical-circuit simulators. Within the framework of this model we show that the coupling is quantified by a mutual inductance extracted by analogy with an ideal transformer. As a consequence optimization guidelines have been defined for the coupling in term of number of loops, position, depending on the bandwidth of the signal to be coupled. Finally the model is used in a case study to evaluate the perturbation brought on the supply line of a BiCMOS technology.

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